IDENTIFICATION OF THE DUAL GROUP

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The goal of this notes is to explain how our knowledge from previous talks about the Satake category are assembled to complete the proof of the geometric Satake equivalence, identifying the Satake category of a reductive group G with the representation category of the Langlands dual group $\check{G}$. This process is called "identification of the dual group" as in the title. The word "identification" is slightly inaccurate since the proof in [2] also exhibits the existence of Chevalley group scheme over $\mathbb{Z}$ with given room datum. This notes will follow the arguments in [1], where the proof if simplified by using the existence theorem of Chevalley group schemes.

1. RECOLLECTION OF THE SATAKE CATEGORY

In this section, we collect our knowledge from previous talks about the Satake category.

Let G be a reductive group over an algebraically closed field k and fix a maximal torus T of G and a Borel subgroup B containing T. Recall some group theory notations: weights $X^*(T)$ are defined as characters on T and the choice of B determines the subset $X^*(T)^+$ of dominant weights, there's a partial order $\leq$ on $X^*(T)^+$ by considering $\mathbb{Z}_{\geq 0}$-combinations of simple roots and a distinguished element 2ρ given by summing positive roots; similarly we have coweights $X_*(T)$ defined as cocharacters on T, dominant coweights $X_*(T)^+$, partial order $\leq$ and distinguished element $2\check{\rho}$; the Weyl group W of G contains a longest element w_0.

We fix a sheaf theory Shv[1]over k with coefficient ring Λ. Recall that the Satake category $\mathcal{A} := \mathrm{Sat}_{G,\Lambda}$ is defined as the abelian category of L^+G-equivariant flat perverse sheaves on Gr_G (or flat perverse sheaves on the Hecke stack $\mathscr{H} = BL^+G \times_{BLG} BL^+G$). We recall it's properties below.

1. (1) There's a natural symmetric monoidal structure $\otimes$ on this category, given by convolution/fusion product. Moreover, the monoidal structure is rigid: for any object $V \in \mathcal{A}$, the dual is given by $X^\vee = sw^*(\mathbb{D}V)$. Here sw is the automorphism of $\mathscr{H}$ given by swapping two BL^+G-factors.
2. (2) There's a fiber functor $F\colon \mathcal{A} \to \mathrm{Vect}_\Lambda$ with target category Vect_Λ being the category of finite projective Λ-modules. The fiber functor is symmetric monoidal[2], exact, conservative and admits a refinement landing in graded finite projective Λ-modules.

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[1]By a sheaf theory we mean either étale, de Rham or Betti sheaves. For any field k, we can use étale sheaves with $\Lambda = \mathbb{Z}_\ell, \mathbb{Q}_\ell, \mathbb{F}_\ell$ for a prime number l coprime to $\mathrm{char}(k)$. If $\mathrm{char}(k) = 0$, we can use de Rham sheaves (or D-modules) with $\Lambda = k$. If $k = \mathbb{C}$, we can use Betti sheaves and take Λ to be any commutating ring.

[2]The fiber functor comes from taking cohomology, so it's a priori symmetric monoidal up to Koszul sign rule. To make F truly symmetric monoidal, one may twist the commutativity constraint of either the source or the target or use super variants instead.

(3) For a standard parabolic subgroup P with Levi quotient M, there's a symmetric monoidal constant term/hyperbolic localization functor $\mathrm{CT}_P\colon \mathcal{A} \to \mathrm{Sat}_{M,\Lambda}$, compatible with fiber functors: F is canonically isomorphic to the composition of CT_P with the fiber functor of M. Constant terms functor are also compatible with composition.

(4) Recall Schubert cells $j_\mu\colon \mathscr{H}_\mu \hookrightarrow \mathscr{H}, \mu \in X_*(T)^+$, we then have costandard, simple and standard objects:

$$\Delta_\mu \to L_\mu \to \nabla_\mu := ({}^p\mathcal{H}^0 j_{\mu,!} \to j_{\mu,!*} \to {}^p\mathcal{H}^0 j_{\mu,*})(\Lambda[\dim \mathscr{H}_\mu]) \in \mathcal{A}.$$

Evaluating constant term functor, $\mathrm{CT}_B(?_\mu)$ is supported on components indexed by the convex hull of $W \cdot \mu$ and equals to Λ on the μ-component, where $? \in \{\Delta, L, \nabla\}$. Consider full subcategories $\mathcal{A}_{\leq\mu} \subset \mathcal{A}$ of sheaves supported on $\mathscr{H}_{\leq\mu} = \bigsqcup_{\lambda\leq\mu} \mathscr{H}_\lambda$, then we have $\mathcal{A} = \mathrm{colim}_\mu \mathcal{A}_{\leq\mu}$.

(5) Suppose $\mathbb{Q} \subset \Lambda$, then the Satake category $\mathcal{A}$ is semisimple. In this case, there's no difference between costandard/simple/standard objects

$$\Delta_\mu \xrightarrow{\simeq} L_\mu \xrightarrow{\simeq} \nabla_\mu.$$

2. CONSTRUCTION OF THE DUAL GROUP

Proposition 2.1. *For $\mu \in X_*(T)^+$, the restriction of F to $\mathcal{A}_{\leq\mu}$ admits a left adjoint $(F|_{\mathcal{A}_{\leq\mu}})^L$. Evaluating at the monoidal unit Λ, we obtain a projective generator P_μ of $\mathcal{A}_{\leq\mu}$.*

Proof. We only present the proof in the case of Betti sheaves and we can reduce to the case $\Lambda = \mathbb{Z}$ by base change. Consider induction on the stronger claim that replaces $\mathcal{A}_{\leq\mu}$ by full categories $\mathcal{A}_I$ of $\mathcal{A}$ supported on I, for finite subsets $I \subset X_*(T)^+$ such that if $\lambda \leq \mu \in I$, then $\lambda \in I$. Enlarging categories, the left adjoint exists by adjoint functor theorem, it suffices to show the resulting projective generator P_I is in the Satake category. A priori, P_I is a sheaf supported on $\mathscr{H}_I = \bigsqcup_{\lambda\in I} \mathscr{H}_\lambda$ and in the heart of perverse t-structure. Pick a maximal element $\mu \in I$ and consider the map ${}^p\mathcal{H}^0 j_{\mu,!} j_\mu^* P_I \to P_I$. We make the following observations:

• $j_\mu^* P_I$ is of the form $M[\dim \mathscr{H}_\mu]$ for a Λ-module M. Evaluate F at ∇_μ, we get

$$F(\nabla_\mu) = \mathrm{Hom}(P_I, \nabla_\mu) = \mathrm{Hom}(j_\mu^* P_I, \Lambda[\dim \mathscr{H}_\mu]) = \mathrm{Hom}(M, \Lambda).$$

Since $F(\nabla_\mu) \in \mathrm{Vect}_\Lambda$, we know M is also a finite projective Λ-module and ${}^p\mathcal{H}^0 j_{\mu,!} j_\mu^* P_I = F(\nabla_\mu) \otimes \Delta_\mu$ is in the Satake category.

• The cokernel C of the map coincides with $P_{I\setminus\{\mu\}}$, since it is supported on $\mathscr{H}_{I\setminus\{\mu\}}$ and corepresents F on $\mathcal{A}_{I\setminus\{\mu\}}$ by the exact sequence

$$0 \to \mathrm{Hom}(C, V) \to \mathrm{Hom}(P_I, V) \to \mathrm{Hom}({}^p\mathcal{H}^0 j_{\mu,!} j_\mu^* P_I, V) = 0, \forall V \in \mathcal{A}_{I\setminus\{\mu\}}.$$

By induction hypothesis, we know $C = P_{I\setminus\{\mu\}}$ is in the Satake category.

• The kernel of the map vanishes. Here we need the technical input from generic fiber and the assumption $\Lambda = \mathbb{Z}$. Since there's also a map $P_I \to {}^p\mathcal{H}^0 j_{\mu,*} j_\mu^* P_I$ and $j_\mu^* P_I$ is constant perverse, it suffices to show $\Delta_\mu \to \nabla_\mu$ is injective. This is true because we have $\Delta_\mu \otimes_\mathbb{Z} \mathbb{Q} \xrightarrow{\simeq} \nabla_\mu \otimes_\mathbb{Z} \mathbb{Q}$ by and the exact conservative fiber functor F valued in free $\mathbb{Z}$-modules. □

Remark 2.2. The fiber functor $\mathrm{Rep}(\check{G})$ forgetting the group action is corepresented by the structure sheaf $\mathcal{O}(\check{G})$ of $\check{G}$. The proof above is about constructing finite truncations of the object $\mathcal{O}(\check{G})$ and proving Peter-Weyl theorem.

Now Tannakian reconstruction is available for the fiber functor $F\colon \mathcal{A} \to \mathrm{Vect}_\Lambda$. The isomorphism $F(P_\mu) \cong \mathrm{End}(P_\mu)^{\mathrm{op}}$ equips an algebra structure on $F(P_\mu)$, giving equivalences $\mathcal{A}_{\leq\mu} \cong F(P_\mu)\text{-mod}(\mathrm{Vect}_\Lambda) \cong F(P_\mu)^\vee\text{-comod}(\mathrm{Vect}_\Lambda)$ and $\mathcal{A} = \mathrm{colim}_\mu \mathcal{A}_{\leq\mu} \cong (\mathrm{colim}_\mu F(P_\mu)^\vee)\text{-comod}(\mathrm{Vect}_\Lambda)$. For $\lambda, \mu \in X_*(T)^+$, the convolution product $P_\lambda \otimes P_\mu$ is in $\mathcal{A}_{\lambda+\mu}$, giving a $F(P_{\lambda+\mu})$-module structure on $F(P_\lambda \otimes P_\mu) = F(P_\lambda) \otimes F(P_\mu)$ and thus a map of algebras $F(P_{\lambda+\mu}) \to F(P_\lambda) \otimes F(P_\mu)$. Dualize, the maps $F(P_\lambda)^\vee \otimes F(P_\mu)^\vee \to F(P_{\lambda+\mu})^\vee$ combines to a commutative algebra structure on $\mathrm{colim}_\mu F(P_\mu)^\vee$ and we denote the corresponding affine scheme by $\check{G}$. Notice $F(P_\mu)^\vee = F(P_\mu^\vee) = \mathrm{Hom}(P_\mu \otimes P_\mu, \mathbb{1})$, swapping two factors gives us an involution S on $\mathcal{O}(\check{G})$ and in fact a Hopf algebra structure on $\mathcal{O}(\check{G})$. In the opposite category, we get an affine flat group scheme $\check{G}$.

The geometric Satake equivalence is then translated into the following theorem.

Theorem 2.3. *$\check{G}$ is isomorphic to the Langlands dual group of G over Λ.*

We observe the following reduction can be made.

Lemma 2.4. *Suppose Theorem 2.3 holds for the adjoint group G_{ad}, then it also holds for G.*

Proof. Observe that $\mathrm{Gr}_G \cong \pi_1(G) \times_{\pi_1(G_{\mathrm{ad}})} \mathrm{Gr}_{G_{\mathrm{ad}}}$. For a finitely generated abelian group A, we define its Cartier dual to be $A^\vee := \mathscr{H}om(A, \mathbb{G}_m)$. Then Cartier duality gives us an t-exact equivalence $\mathrm{Shv}(A_k) \cong \mathrm{QCoh}(A_\Lambda) \cong \mathrm{QCoh}(BA_\Lambda^\vee) = \mathrm{Rep}(A_\Lambda^\vee)$. From this equivalence and Theorem 2.3 for G_{ad} we get

$$\mathrm{Sat}_{G,\Lambda} \cong \mathcal{O}((\pi_1(G))^\vee \times^{\pi_1(G_{\mathrm{ad}})^\vee} \check{G_{\mathrm{ad}}})\text{-comod}(\mathrm{Vect}_\Lambda),$$

hence an isomorphism $\check{G} \cong \pi_1(G))^\vee \times^{\pi_1(G_{\mathrm{ad}})^\vee} \check{G_{\mathrm{ad}}}$, identified with the Langlands dual group of G as desired. □

3. CASE $G = \mathrm{PGL}_2$

We start with the case $G = \mathrm{PGL}_2$. Then there's a order-preserving bijection between dominant coweights and non-negative integers. Since $\mathrm{Gr}_{\mathrm{PGL}_2,1} \cong \mathbb{P}^1$, there's a $\check{G}$ action on $F(L_\mu) = \Lambda^{\oplus 2}$ and thus a map $\check{G} \to \mathrm{GL}_2$. On the other hand, the constant term functor CT_B gives us a map $\check{T} \to \check{G}$, which is a closed immersion because CT_B generates the target category by the control of weight spaces of $\mathrm{CT}_B(L_\mu)$.

We first consider the characteristic zero case that assuming $\mathbb{Q} \subset \Lambda$. We may assume Λ is a field by base change. We claim the following properties of $\check{G}$.

$\check{G}$ is of finite type. This is translated to the claim of $\mathcal{A}$ that there exists a tensor generator. The claim can be verified by taking the direct sum of L_μ with μ running over a finite generating subset of $X_*(T)^+$ and using semisimplicity.

$\check{G}$ is connected. Given the previous claim, this is translated to the claim of $\mathcal{A}$ that there doesn't exist a finitely generated subcategory closed under $\otimes$. The claim can be verified by looking at highest weights and using that CT_B is monoidal.

$\check{G}$ is reductive. Given previous claims, this is translated to the claim that $\mathcal{A}$ is semisimple, which we already know.

$\check{T}$ is a maximal torus of $\check{G}$. Given previous claims, we need to compute the rank of $\check{G}$. Given a finite set of tensor generators of a reductive group, the number of simple objects generated by tensor products of no more than n elements in the finite set grows as a polynomial in n of degree equals to the rank of the reductive group. This characterization ensures that the rank of $\check{G}$ equals to the rank of G.

In conclusion, $\check{G}$ is a rank 1 split reductive group which admits a map to GL_2, thus $\check{G}$ has to be SL_2 and the map to GL_2 is identified with the natural inclusion.

We move back to the case of general Λ, where we can assume $\Lambda = \mathbb{Z}$ in the Betti case and $\Lambda = \mathbb{Z}_\ell$ in the étale case. We only treat the later one to save notations. Then the map $\check{G} \to \mathrm{GL}_2$ factors through the closed subgroup SL_2 of GL_2 since it holds on the generic fiber.

We want to control the special fiber $\check{G}_{\mathbb{F}_\ell}$. The representation category of $\check{G}_{\mathbb{F}_\ell}$ is not semisimple, we know neither finiteness nor connectedness of $\check{G}_{\mathbb{F}_\ell}$. We only have control on its simple objects. The idea to resolve the situation is to approximate $\check{G}_{\mathbb{F}_\ell}$ by an algebraic group of finite type, so we consider the schematic image H of $\check{G}_{\mathbb{F}_\ell}$ in $\mathrm{SL}_{2,\mathbb{F}_\ell}$. Since H is of finite type, we can show connectedness of H by highest weights as before. As an algebraic group in positive characteristic, H is not necessarily smooth. However, we have the following Frobenius trick to erase the difference for our purpose: the reduced subscheme H_{red} naturally inherits a group scheme structure and is smooth. The Frobenius map $\mathrm{Fr}_\ell \colon H \to H$ is a group homomorphism and note Fr_ℓ^N factors through H_{red} for N sufficiently large as the ideal sheaf defining H_{red} consists of nilpotent elements and H is of finite type. Moreover, the map $(\mathrm{Fr}_\ell^N)^*\mathcal{O}(H_{\mathrm{red}}) \to \mathcal{O}(H)$ is injective. Therefore, we get injections $\mathcal{O}(H_{\mathrm{red}}) \hookrightarrow \mathcal{O}(H) \hookrightarrow \mathcal{O}(\check{G})$ of coalgebras and hence also injections on the set of simple objects in the comodule category. Now we use $G = \mathrm{PGL}_2$. The simple objects of $\check{G}$ are indexed by $\mathbb{Z}_{\geq 0}$. H_{red}, as a smooth connected closed subgroup of SL_2, is either isomorphic to $\mathbb{G}_m, \mathbb{G}_m \ltimes \mathbb{G}_a$ or the whole SL_2. Then the injection on simple objects forces H to be SL_2, i.e. $\check{G}_{\mathbb{F}_\ell} \to \mathrm{SL}_{2,\mathbb{F}_\ell}$ is surjective.

Finally, the information on generic and special fibers is enough to show $\check{G} \to \mathrm{SL}_2$ is an isomorphism, by the following lemma.

Lemma 3.1. *A map of flat $\mathbb{Z}_\ell$-modules $f \colon M \to N$ is isomorphism if $f \otimes \mathbb{Q}_\ell$ is an isomorphism and $f \otimes \mathbb{F}_\ell$ is injective.*

We conclude that Theorem 2.3 holds for $G = \mathrm{PGL}_2$. By Lemma 2.4, we also conclude that Theorem 2.3 holds for any reductive group G of semisimple rank 1.

4. Proof in the general case

For a general reductive group G, it's helpful to also utilize more constant term functors. For a simple root α, consider the corresponding parabolic subgroup P_α and Levi subgroup M_α. Then CT_{P_α} and the constant term functor of M_α give us maps $\check{T} \to \check{M}_\alpha \to \check{G}$. Note $\check{M}_\alpha$ is already identified with the Langlands dual group of M_α since it has semisimple rank one.

We again start with the case $\mathbb{Q} \subset \Lambda$ and Λ is a field. The same reasoning as before shows that $\check{G}$ is of finite type, connected and finally split reductive with rank as expected. Denote $\check{W}$ the Weyl group of $\check{G}$. Taking Lie algebra of the maps $\check{M}_\alpha \to \check{G}$, we realize $\alpha, \check{\alpha}$ and s_α as coroots, roots and simple refelections in the Weyl group of $\check{G}$ for all simple roots α. Since both W and $\check{W}$ are reflection groups and a set of generators of W is contained in $\check{W}$, W is realized as a subgroup of $\check{W}$. However, the weight space of L_μ, being invariant under the $\check{W}$-action, is contained in the convex hull of $W \cdot \mu$, forcing the equality $W = \check{W}$. Together with the known roots/coroots of $\check{G}$ obtained from $\check{M}_\alpha$, we can identify the root datum of $\check{W}$ with the Langlands dual room datum of G, finishing the proof of Theorem 2.3 in the characteristic zero case.

For general coefficients Λ, we can reduce to the case $\Lambda = \mathbb{Z}_\ell$ by base change and Proposition 1.5 in [3]. We nwo use the existence of the Langlands dual group $\hat{G}$ of G over $\mathbb{Z}_\ell$. By the characteristic zero case, there's an isomorphism $\check{G}_{\mathbb{Q}_\ell} \simeq \hat{G}_{\mathbb{Q}_\ell}$ compatible with the maps $\check{T} \to \check{M}_\alpha \to \check{G}$ and $\check{T} \to \check{M}_\alpha \to \hat{G}$ coming from constant term functors and duality.

Let $\breve{\mathbb{Q}}_\ell$ be the (completed) maximal unramified extension of $\mathbb{Q}_\ell$ and $\breve{\mathbb{Q}}_\ell$ be its ring of integers. Consider $\breve{\mathbb{Z}}_\ell$-points of $\check{G}$ and $\hat{G}$ as subsets of $\check{G}(\breve{\mathbb{Q}}_\ell) \simeq \hat{G}(\breve{\mathbb{Q}}_\ell)$, then the images of $\check{M}_\alpha(\breve{\mathbb{Z}}_\ell)$ in them coincide. Since $\check{G}(\breve{\mathbb{Z}}_\ell)$ is generated (not topologically!) by the $\breve{\mathbb{Z}}_\ell$-points of the Levi $\check{M}_\alpha$'s, we get an inclusion $\hat{G}(\breve{\mathbb{Z}}_\ell) \subset \check{G}(\breve{\mathbb{Z}}_\ell)$.

The inclusion of points implies $\mathcal{O}(\check{G}) \subset \mathcal{O}(\hat{G})$ as subsets of $\mathcal{O}(\check{G}_{\mathbb{Q}_\ell}) \simeq \mathcal{O}(\hat{G}_{\mathbb{Q}_\ell})$. Indeed, suppose we have $f \in \mathcal{O}(\check{G})$ with $\ell f \in \mathcal{O}(\hat{G})$ nonzero modulo ℓ, then there exists a point $\overline{x} \in \hat{G}(\overline{\mathbb{F}}_\ell)$ such that $\ell f(\overline{x}) \neq 0$ by smoothness of $\hat{G}$. Lift $\overline{x}$ to $x \in \hat{G}(\breve{\mathbb{Z}}_\ell) \subset \check{G}(\breve{\mathbb{Z}}_\ell)$, we get $0 = \ell(f(x)) = (\ell f)(x) \neq 0 \mod \ell$, a contradiction.

The inclusion of functions is automatically an injection of Hopf algebras, so we get a group homomorphism $\hat{G} \to \check{G}$. To control the finiteness of $\check{G}$, we approximate it by choosing a representation of $\check{G}$ on finite free $\Lambda = \mathbb{Z}_\ell$-module V being faithful on the generic fiber. If we can show the composition $\hat{G} \to \check{G} \to \mathrm{GL}(V)$ is a closed immersion, then $\check{G} \to \mathrm{GL}(V)$ factors through the subgroup $\hat{G}$, exhibiting $\check{G}$ as a retract of $\hat{G}$. Then $\mathcal{O}(\check{G}) \subset \mathcal{O}(\hat{G})$ forces $\hat{G} \to \check{G}$ to be an isomorphism. By the following lemma, we do have a closed immersion for groups G of adjoint type. By Lemma 2.4, this finishes the proof of Theorem 2.3.

Lemma 4.1 ([3],Corollary 1.3)**.** *Let G be a reductive group over $\mathbb{Z}_\ell$. Assume either $l \neq 2$ or no normal algebraic subgroup of $G_{\overline{\mathbb{Q}}_\ell}$ is isomorphism to $\mathrm{SO}_{2n+1}, n \geq 1$. Let $\phi\colon G \to H$ be a morphism of affine group schemes of finite type over $\mathbb{Z}_\ell$ such that $\phi_{\mathbb{Q}_\ell}\colon G_{\mathbb{Q}_\ell} \to H_{\mathbb{Q}_\ell}$ is a closed immersion. Then ϕ is a closed immersion.*

The strange conditions in the lemma comes from the following exotic examples in characteristic 2.

Example 4.2. Set $\Lambda = \mathbb{F}_2$, there's a unipotent isogeny $\mathrm{SO}_{2n+1} \to \mathrm{Sp}_{2n}$. In fact, the notion of quadratic forms over $\mathbb{F}_2$ is no longer the same as symmetric bilinear forms. On the vector space $\mathbb{F}_2^{2n+1}$ with basis $e_{-n}, e_{1-n}, ...e_n$, the quadratic form $Q(\sum_{i=-n}^n x_i e_i) = x_0^2 + \sum_{i=1}^n x_i x_{-i}$ is non-degenerate, and we denote the subgroup of GL_{2n+1} preserving the quadratic form by SO_{2n+1}. Then SO_{2n+1} also preserves the symmetric pairing $B(v, w) = Q(v+w) - Q(v) - Q(w)$, which has a 1-dimensional radical and induces a symplectic pairing on the $2n$-dimensional quotient. This gives us the unipotent isogeny $\mathrm{SO}_{2n+1} \to \mathrm{Sp}_{2n}$.

Example 4.3. Set $\Lambda = \mathbb{Z}_2[\sqrt{2}]$. Consider the $\Lambda[\frac{1}{2}]$-vector space $V = \Lambda[\frac{1}{2}]^{2n+1}$ with basis $e_{-n}, e_{1-n}, ...e_n$ and the quadratic form $Q(\sum_{i=-n}^n x_i e_i) = x_0^2 + \sum_{i=1}^n x_i x_{-i}$. Consider two lattices in V: $L = \Lambda^{2n+1}$ and $L' = \Lambda \cdot x_0/\sqrt{2} + L$. Define affine flat group schemes G and G' over Λ as the schematic image of $\mathrm{SO}(V, Q)$ in $\mathrm{GL}(L)$ and $\mathrm{GL}(L')$ respectively. In fact, the G-action on V preserves the lattice L' and defines maps $G \to G' \to \mathrm{GL}(L')$. G is the Chevalley group scheme SO_{2n+1} and G' is a quasi-reductive group scheme with generic fiber SO_{2n+1} and reduced special fiber Sp_{2n}.

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