Seminar on Bezrukavnikov's Equivalence

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Abstract

The goal of the seminar is to cover the proof of the geometric Satake equivalence and Arkhipov–Bezrukavnikov's equivalence, which are two primitive versions of the categorical local Langlands conjecture.

1 Introduction

Let F be a local field with residue characteristic p and ring of integers $\mathcal{O}$, with residue field k of size q. Let $G/\mathcal{O}$ be a split reductive group. The Local Langlands Correspondence (LLC) attempts to understand smooth representations of $G(F)$ in terms of Galois representations into the dual group $\check{G}$. Although the correspondence is still conjectural in general, those representations V such that $V^{G(\mathcal{O})} \neq 0$ is well understood. Such representations are in bijection with modules over the spherical Hecke algebra $\mathcal{H}_{\mathrm{sph}} := \mathbb{C}[G(\mathcal{O})\backslash G(F)/G(\mathcal{O})]$, which admits the following description.

Proposition 1.1 (Satake). *There is an isomorphism*

$$\mathcal{H}_{\mathrm{sph}} \simeq R(\check{G}), \tag{1}$$

where $R(\check{G})$ is the representation ring of $\check{G}$.

As a consequence, we deduce the unramified LLC.

Corollary 1.2. *There is a bijection between:*

- *irreducible representations of $G(F)$ with a $G(\mathcal{O})$-fixed vector; and*
- *semisimple elements $s \in \check{G}$ up to conjugation (the* Satake *parameter).*

Under the philosophy of geometric Langlands, we hope to find a categorical equivalence which recovers (1) upon passing to K-groups. A natural categorification of the left-hand side (at least in the equal characteristic case) is the category of perverse sheaves

$$\mathrm{Perv}(L^+G\backslash LG/L^+G),$$

where $LG(R) := G(R(\!(t)\!))$ is the loop group and $L^+G(R) := G(R[\![t]\!])$ is the positive loop group. On the other hand, the right-hand side naturally categorifies to the category of representations $\mathrm{Rep}(\check{G})$. Thus we would hope for an equivalence between these two categories. In fact, since (1) is an isomorphism of *commutative rings*, not just vector spaces, we hope that both categories carry a symmetric monoidal structure and the equivalence is symmetric monoidal. The Geometric Satake equivalence realizes our hope.

Theorem 1.3 (Mirković-Vilonen [24], Ginzburg [20], Lusztig)**.** *There is a symmetric monoidal equivalence*

$$\mathrm{Perv}(L^+G\backslash LG/L^+G) \simeq \mathrm{Rep}(\check{G}), \tag{2}$$

where the left-hand side carries the convolution monoidal structure and the right-hand side carries the usual monoidal structure.

Remark. The theorem provides a "canonical" definition of the Langlands dual $\check{G}$, rather than via the combinatorics of root datum.

Now that we know what the representations with a $G(\mathcal{O})$-fixed vector look like, the next natural goal is to take a smaller subgroup $I \subset G(\mathcal{O})$ and consider representations with I-fixed vectors. We let $I \subset G(\mathcal{O})$ be the *Iwahori subgroup*, i.e., the pre-image of a Borel subgroup $B(k)$ under the reduction map $G(\mathcal{O}) \to G(k)$. Again such representations are controlled by the Iwahori-Hecke algebra $\mathcal{H}_I := \mathbb{C}[I\backslash G(F)/I]$.

Proposition 1.4 (Kazhdan-Lusztig '87 [22])**.** *There is a $\check{G}$-variety* St*, called the* Steinberg variety*, and there exists a ring isomorphism*

$$\mathcal{H}_I \simeq K^{\check{G}\times\mathbb{G}_m}(\mathrm{St}) \otimes_{K^{\mathbb{G}_m}(\mathrm{pt})} \mathbb{C}_q, \tag{3}$$

where $K^{\check{G}\times\mathbb{G}_m}(\mathrm{St})$ denotes the $\check{G}\times\mathbb{G}_m$-equivariant K-theory of St*.*

As a consequence, we deduce the Deligne-Langlands correspondence.

Corollary 1.5. *There is a bijection between:*

- *irreducible representations of $G(F)$ with an I-fixed vector; and*
- *a tuple (s, u, χ) of a semisimple element $s \in \check{G}$ and a unipotent element $u \in \check{G}$ such that $sus^{-1} = u^q$ and an irreducible representation χ of the centralizer $\pi_0 \mathrm{Z}_{\check{G}}(s, u)$, all up to $\check{G}$-conjugation.*

Again we hope to categorify the isomorphism (3), so the left-hand side naturally categorifies to $\mathrm{Perv}(I\backslash LG/I)$ where I is the pre-image of $B \subset G$ under $L^+G \to G$ and the right-hand side naturally categorifies to the category of $\check{G}$-equivariant coherent sheaves $\mathrm{Coh}^{\check{G}}(\mathrm{St})$. We thus expect, in analogy with Theorem 1.3, an equivalence

$$\mathrm{Perv}(I\backslash LG/I) \simeq \mathrm{Coh}^{\check{G}}(\mathrm{St}).$$

This turns out to be too good to be true. However, such an equivalence *does* hold if we work on the derived level.

Theorem 1.6 (Bezrukavnikov [8]). *There is a monoidal equivalence of triangulated categories (or ∞-categories)*

$$\mathcal{D}_I := D^b_{\mathrm{cons}}(I\backslash LG/I) \simeq D^b_{\mathrm{coh}}(\check{G}\backslash \mathrm{St}).$$

Here, it is important that St is considered as a *derived* scheme.

The proof of this equivalence is based on a result due to Arkhipov–Bezrukavnikov, which categorifies the aspherical module of $\mathcal{H}_I$ in terms of (equivariant) coherent sheaves on the Springer variety $\widetilde{\mathcal{N}}$. This is known as the Arkhipov–Bezrukavnikov equivalence.

Theorem 1.7 (Arkhipov–Bezrukavnikov [5]). *There is a quotient ${}^f\mathcal{D}_I$ of $\mathcal{D}_I$ and an equivalence*

$${}^f D_I \simeq D^b_{\mathrm{coh}}(\check{G}\backslash\widetilde{\mathcal{N}})$$

The goal of the seminar is to cover the proofs of Theorem 1.3 and Theorem 1.7. If time and participants permit, we will discuss Theorem 1.6 in a suitable follow-up format (workshop or seminar).

Time and location: Fridays 2-4 PM at Simonyi 101, IAS, with the exceptions on Mar 6, 20, Apr 3, 10, where the talks are at Bloomberg (physics) Lecture Hall.

2 Talks

2.1 Representation-theoretic background, 2/6

Briefly review combinatorics and algebraic representation of reductive groups: root systems, Weyl groups and (extended) affine Weyl groups, weight spaces, classification of irreducible representations in terms of highest weights etc.. Define the Langlands dual group. Review p-adic groups and their (smooth) representations, including: Hecke algebras attached to compact open subgroups, relation between representations and Hecke modules, classical Satake isomorphism and its proof (sketch). If time permits, discuss the unramified local Langlands correspondence.

References: [29, Talk 1-2], [13, Chapter 1].

2.2 Geometry of affine Grassmannian and flag varieties, 2/13

Introduce the affine Grassmannian and affine flag varieties. Discuss their geometry, in particular the Bruhat decomposition (e.g. mention the dimensions of Schubert cells and notice the parity) and semi-infinite orbits. Focus on the example of SL_2.

References: [6, §3], [16, §2], [24, Proposition 3.1, Theorem 3.2].

2.3 Geometric Satake I: The Satake category, 2/20

Briefly review (equivariant) perverse sheaf theory and Tannakian formalism. Define the Satake category with the convolution product and fiber functor. State the geometric Satake equivalence. In particular, prove convolution of perverse sheaves is perverse [2, Lemma 1.3.2], [24, Proposition 4.2].

References: [15], [2, §1.3.1-1.3.2]. For some historical overview, see [2, §0.1.2-0.1.3].

2.4 Geometric Satake II: The commutativity constraint, 2/27

Classically, Satake proved that the convolution product on the spherical Hecke algebra is commutative. On the categorical level, this corresponds to the fact that for sheaves $\mathcal{F}$ and $\mathcal{G}$ in the Satake category, there is a natural isomorphism

$$\mathcal{F} \star \mathcal{G} \simeq \mathcal{G} \star \mathcal{F}. \tag{4}$$

To establish this, we re-interpret the convolution product by considering families of affine Grassmannians. For a curve X, Beilinson-Drinfeld defined an ind scheme Gr_{G,X^2} over X^2, whose fibers over the diagonal $\Delta(X)$ are Gr_G and whose fibers over the open complement $X^2 \backslash \Delta(X)$ are $\mathrm{Gr}_G \times \mathrm{Gr}_G$. The fusion product *is defined by considering the sheaf $\mathcal{F} \boxtimes \mathcal{G}$ on $\mathrm{Gr}_{G,X^2}|_{X^2\backslash\Delta(X)}$, taking an IC extension to Gr_{G,X^2}, and restricting to the diagonal. From this perspective, X^2 has an obvious symmetry permuting the factors, giving rise to the commutativity constraint* (4)*.*

Introduce the Beilinson–Drinfeld affine Grassmannian. Prove that the convolution product is commutative via fusion. If time permits, explain the construction of convolution via nearby cycles and compare the two constructions as in [2, §3.3.2].

References: [6, §7.5], [2, §1.3.3, §3.3.1-2], [24, §5]

2.5 Geometric Satake III: Constant Term functors, 3/6

Define Braden's hyperbolic localization functor and prove [11, Theorem 1]. Introduce constant term functors and discuss their properties. Discuss the cohomology of semi-infinite orbits and the grading on the fiber functor. Thus prove the properties of the fiber functor stated in [2, §1.3.2].

References: [11], [25], [2, §1.1, 1.3.2]

2.6 Geometric Satake IV: Identification of the dual group, 3/13

Using the Tannakian formalism, the Satake category identifies with the representation category of an algebraic group H. We translate the geometric input from the last few talks to study the root system of H and eventually identify it with the Langlands dual group $\check{G}$. Focus on the SL_2 case.

References: [2, §1.4], [17, §VI.11].

2.7 Geometric Satake V: Derived Satake, 3/20

Geometric Satake gives an equivalence of abelian *categories*

$$\mathrm{Sat}(G) := \mathrm{Perv}(L^+G\backslash LG/L^+G) \simeq \mathrm{Rep}(\check{G}).$$

The derived Satake equivalence upgrades this to the derived category of constructible sheaves:

Theorem 2.1 (Bezrukavnikov-Finkelberg)**.** *There is a monoidal equivalence*

$$D^b_{\mathrm{cons}}(L^+G\backslash LG/L^+G) \simeq D^b_{\mathrm{coh}}((0 \times^{\mathbb{L}}_{\check{\mathfrak{g}}} 0)/\check{G}).$$

State the derived Satake equivalence. Sketch the proof with focus on the computation for SL_2.

References: [9] (§5 is the rank 1 computation).

2.8 Iwahoric Hecke algebra and their representations, 3/27

Introduce Iwahori subgroups, Iwahori-Hecke algebras and discuss its relation to affine Hecke algebra. Discuss its Bernstein presentation, which in particular identifies its center with the spherical Hecke algebra (Bernstein isomorphism). Define the subalgebra A. Introduce the notion of Whittaker models of a (generic) representation of a p-adic group (sketch a proof of its uniqueness if time permits) and provide some motivation. Define the aspherical module and its three incarnations [5, §1.1.1]. Give examples in the GL_2 case.

References: [2, §5.1], [14, §7.1]. For some motivation of Whittaker models, see for example notes by T. Feng [18] or [12, §3.5].

2.9 Springer and Steinberg varieties, 4/3

Introduce the Springer variety $\widetilde{\mathcal{N}}$, the basic affine space $\check{G}/\check{U}$ and its affine completion, the affine completion of the unipotent Springer variety[1]. Study (equivariant) coherent sheaves on the Springer variety, in particular, prove [2, Proposition 6.2.10], [5, Lemma 21]. Describe $K(D^{\check{G}\times\mathbb{G}_m}(\widetilde{\mathcal{N}}))$ in terms of the aspherical module. Discuss the example of SL_2.

If time permits, briefly introduce the Grothendieck-Springer variety $\widetilde{\mathfrak{g}}$, the Steinberg variety St and state the Kazhdan–Lusztig isomorphism

$$\mathcal{H}_q(W) \simeq K(D^{\check{G}\times\mathbb{G}_m}(\mathrm{St})).$$

References: [2, §6.2], [14, §7.5], [27, Lecture 22]

[1] Denoted $\hat{\mathcal{N}}_{\mathcal{X}}$ in [2] and $\hat{\tilde{\mathcal{N}}}_{af}$ in [5].

2.10 Gaitsgory's central functor I: Construction, 4/10

Discuss the degeneration of affine Grassmannian into flag varieties as in [2, §2.2.3], as well as the Beilinson–Drinfeld version. Review nearby cycles and their monodromy automorphism [2, §9.1.3]. Construct Gaitsgory's central functor $\mathcal{Z}$ and prove its basic properties (t-exactness [2, Lemma 2.4.5], carrying unipotent monodromy [2, Proposition 2.4.6] etc.).

References: [2, §2, §9], [21]

2.11 Gaitsgory's central functor II: Centrality, 4/17

The analog of Bernstein's identification of the center of $\mathcal{H}_q(W)$ *with the spherical Hecke algebra will be that for any* $\mathcal{F} \in \mathrm{Sat}(G)$ *and* $\mathcal{G} \in \mathcal{P}_I := \mathrm{Perv}(I\backslash LG/I)$ *there is an isomorphism, natural in* $\mathcal{F}$ *and* $\mathcal{G}$*,*

$$\mathcal{Z}(\mathcal{F}) \star \mathcal{G} \simeq \mathcal{G} \star \mathcal{Z}(\mathcal{F}).$$

We need to construct these centrality isomorphisms.[2]

2.11.1 Centrality

Define central functors. State and prove [2, Theorem 3.1.1, Corollary 3.2.5]. Discuss the compatibility with braid relations [2, Theorem 3.2.3].

References: [2, §3]

2.11.2 Symmetric monoidality

Discuss the monoidality of the central functor [2, §3.4]. Show that the centrality isomorphisms on $\mathcal{Z}$ is compatible with the commutativity constraint on the Satake category [2, Theorem 3.5.1], by comparing the nearby cycle construction of convolution product with fusion.

References: [2, §3.5], [19].

Alternatively, one can organize this talk following [3, §4.2-4.3].

2.12 Wakimoto sheaves and Wakimoto filtration, 4/24

The Wakimoto sheaves on $\mathcal{P}_I$ *categorify the Bernstein generators* $\theta_\lambda \in \mathcal{H}_q(W)$*. The central sheaves have filtration by the Wakimoto sheaves.*

State and prove [5, Theorem 4]. Briefly explain [5, Theorem 6], and compare the grading functor to the Satake fiber functor [2, Equation (4.8.1), Proposition 4.8.2].

References: [3, §3.2], [5, §3.6], [2, §4.1-4.4, 4.7-4.8]

[2] There are two parts of the talk and the speaker could decide to give the details of one part.

2.13 Arkhipov–Bezrukavnikov Equivalence I: Construction, 5/1

Gaitsgory's central functor $\mathcal{Z}$ and the Wakimoto sheaves combine to a functor $\mathrm{Rep}(\check{G} \times \check{T}) \to \mathcal{P}_I$*. By specifying certain compatibilities, this extends uniquely to a monoidal functor from free coherent sheaves on the affine completion* $\hat{\tilde{\mathcal{N}}}_{af}$*, which eventually induces a functor*

$$F : D^b_{\mathrm{coh}}(\check{G} \backslash \widetilde{\mathcal{N}}) \to \mathcal{D}_I := D^b_{\mathrm{c}}(I \backslash LG/I).$$

There is a quotient ${}^f\mathcal{D}_I$ *of* $\mathcal{D}_I$ *that categorifies the aspherical module; Arkhipov-Bezukavnikov states the composition of* F *with* $\mathcal{D}_I \to {}^f\mathcal{D}_I$ *is an equivalence.*

Recall results from Talk 2.9. Discuss the monodromy and highest weight arrows as in [5, §3.3]. Use these to construct the functor $\widetilde{F}$ following [2, §6.3.4-6.3.5] and show it descends to the desired functor F [2, Proposition 6.3.9], cf. [5, §3.4, 3.7]. Define the aspherical quotient of the affine Hecke category and state [5, Theorem 1].

Reference: [2, §6.3], [5, §3]

2.14 Arkhipov–Bezrukavnikov Equivalence II: Whittaker-averaging, 5/4

The aspherical category ${}^f\mathcal{D}_I$ *has a variant* $\mathcal{D}_{IW}$*, the category of Iwahori-Whittaker sheaves, which categorifies the Iwahori-Whittaker model of the aspherical module. The equivalence* ${}^f\mathcal{D}_I \simeq \mathcal{D}_{IW}$ *is induced by an averaging functor* Av_Ψ*, see [5, Theorem 2]. Therefore we are reduced to show that* $F_{IW} := \mathrm{Av}_\Psi \circ F$ *is an equivalence. This turns out easier to prove by the tilting property of images of central sheaves in* $\mathcal{D}_{IW}$*.*

Review Artin-Schreier sheaves. Define the Iwahori–Whittaker category and the averaging functor as in [2, §6.4.2-6.4.3]. Mention [5, Lemma 22]. Prove [5, Proposition 2], cf. [2, Theorem 6.4.2].

References: [2, §6.4], [5, §1.6, 2].

2.15 Arkhipov–Bezrukavnikov Equivalence III: Proof, 5/8

This talk contains two parts and the speaker could choose one part to focus on, or take more than one session if necessary.

2.15.1 Tilting Iwahori-Whittaker perverse sheaves

Briefly review highest-weight categories and tilting objects. Discuss the highest weight structure on the perverse Iwahori–Whittaker sheaves $\mathcal{P}_{IW}$, see [2, §6.4.3], cf. [3, Proposition 6.4]. Prove that the images of central sheaves are tilting, by reducing to the minuscule representations (assume G has a faithful minuscule representation in this talk) following [3, §7], see also [2, Theorem 6.5.2, Proposition 6.5.7, 6.5.9], [5, Proposition 7, Lemma 25, 26].

References: [3, §7], [2, §6.5], [5, §4.4]. For highest-weight categories, see [26, §7] and [23].

2.15.2 Regular quotient and finishing the proof

Introduce the regular quotient of $\mathcal{P}_I$. Show that it inherits a monoidal structure and central functor [2, §6.5.6]. Describe the regular quotient via Tannakian formalism [2, §6.5.7], [2, Proposition 8.3]. Briefly discuss regularity of $\mathfrak{n}_0$ as in [3, Proposition 8.6]. Prove [5, Proposition 8] and thus finish the proof of the Arkhipov–Bezrukavnikov equivalence [5, §4.2.1].

References: [5, §4.2-4.3], [3, §8-9], [2, §6.5.6-6.5.8].

3 Additional topics

Time admitting, we will try to cover the following topics. Depending on interest, we may also hold a week-long workshop at the end of the semester.

3.1 Exotic t-structure on the Springer resolution

Under Arkhipov–Bezrukavnikov's equivalence, the perverse t-structure on the constructible side transports to a t-structure on the coherent side, which is usually referred to as the exotic t-structure.

Define the exotic exceptional collection on $\mathcal{D}^b(\mathrm{Coh}^{\check{G}}(\widetilde{\mathcal{N}}))$ indexed by cocharacters of G as in [2, §7.1.2]. Use this to define the exotic t-structure. Prove that this matches the perverse t-structure under Arkhipov–Bezrukavnikov's equivalence F_{IW} [2, Proposition 7.1.5]. As a consequence, provide a description of the regular quotient category as in [2, §7.2].

References:[3, §10], [2, §7]

3.2 Bezrukavnikov's Equivalence I

There are three versions of Bezrukavnikov's equivalence

$$D_{I_0I_0} \simeq D^b(\mathrm{Coh}^{\check{G}}_{\mathcal{N}}(\mathrm{St})),\ D_{I_0I} \simeq D^b(\mathrm{Coh}^{\check{G}}(\mathrm{St}')),\ D_{II} \simeq \mathrm{DGCoh}^{\check{G}}(\widetilde{\mathcal{N}} \times^{\mathbb{L}}_{\check{\mathfrak{g}}} \widetilde{\mathcal{N}}).$$

The first equivalence formally implies the other two so our main task will be to understand $D_{I_0I_0}$. It turns out that it is more convenient to work with a certain completion $D^{\wedge}_{I_0I_0}$ constructed by Yun.

Review derived schemes and monodromic sheaves, Yun's completion of the category of monodromic sheaves.

References: [29, Talk 16]. For monodromic sheaves [28], see [8, §3]. For free monodromic sheaves, see [10, Appendix A].

3.3 Bezrukavnikov's Equivalence II

Classically, the aspherical quotient of $\mathcal{H}_p(W)$ can be studied via the Schur antisymmetrizer $\xi := \sum_{w\in W}(-1)^{\ell(w)}T_w$. A key ingredient in the proof of Bezrukavnikov's equivalence is the big tilting object $\widehat{\Xi}$ which categorifies the antisymmetrizer.

Construct the big tilting and prove its relation to Iwahori-Whittaker averaging. Sketch the proof of the equivalence.

References: [8, §5]

3.4 Application: Categorical trace and the local Langlands correspondence

One can explore the geometric equivalences to give a categorification of the Deligne-Langlands correspondence by taking the categorical trace of Frobenius, cf. the work of Ben-Zvi–Chen–Helm–Nadler [7] and Zhu [31].

Prove the compatibility with Frobenius [5, Proposition 1], [8, Proposition 53]. Explain the ideas of the formalism of categorical trace [31, §7-8] and its consequences when applied to Bezrukavnikov's equivalence.

References: [31], [5], [8]

4 References

Our main references are: [2], [3], [5], [8]. Other good general references include: [27]. For online workshop programs on the same topic, see [29] program, Seminar program (Lourenço and Zou) [4].

For the geometric Satake equivalence, one can read [20], [24], [1], [30], [16], cf. [17, Chapter VI].[3]

References

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